The behavior of the eigenvalues of large random matrices is generally very predictable, on multiple scales. Macroscopically, results like the semi-circle law describe the overall shape of the eigenvalue distributions, and it is often the case that spectral measures are approximated asymptotically almost surely, and with known estimates on distances, by deterministic limiting measures. On a microscopic scale, we may see the phenomenon of eigenvalue rigidity, in which individual eigenvalues concentrate strongly at predicted locations. I will describe some general approaches to these phenomena, with many examples: Wigner matrices, Wishart matrices, random unitary matrices, truncations of random unitary matrices, Brownian motion on the unitary group, and others.